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Radial heat transfer in packed beds of spheres, cylinders and Rashig rings Verification of model with a linear variation of λ_{er} in the vicinity of the wall

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Abstract

Experimental data on the effective radial thermal conductivities and wall heat transfer coefficients of cylindrical beds formed of spheres, cylinders and Rashig rings are presented. The obtained heat transport parameters are compared with literature data. A model with a linear variation of λ_{er} in the vicinity of the wall is proposed to the description of the radial heat transfer in the packed bed. The model allows simple correlation between the wall Nusselt number and the bed core effective radial thermal conductivity. The model is verified by experiments.

Keywords: Heat transport; Packed bed; Shaped catalyst; Modelling

1. Introduction

For design of tubular packed bed heat exchangers, adsorbers and chemical reactors it is very important to know the mechanism of the radial heat transport from the tube wall to the packed bed. There are two basic types of radial heat transport models. The so-called α_w-models use two parameters for description of the radial heat transport: the wall heat transfer coefficient α_w and the effective radial thermal conductivity λ_{er}. The main disadvantage of these models is the unrealistic fluid temperature near the wall and as a result incorrect prediction of the reaction rates. To avoid this problem varying effective radial thermal conductivity $\lambda_{er}(r)$ and boundary condition of the first kind at the wall are used [1–3]. Such models are called $\lambda_{er}(r)$-models.

At present work, a model with a linear variation of λ_{er} in the vicinity of the wall is proposed. This $\lambda_{er}(r)$-model results in a simple correlation between the wall heat transfer coefficient (in terms of the wall Nusselt number, Nu_w) and the constant effective radial thermal conductivity of the packed bed core.

A common heat exchange type study of heat transfer in packed beds in the steady state and without chemical reaction were carried out to determine the heat transfer parameters α_w and λ_{er} of the standard dispersion model (SDM). Spheres of different sizes, cylinders and Rashig rings with diameter equal to height were used in the experiments.

The dependences of the effective radial thermal conductivity and the wall Nusselt number on the gas velocity (in terms of Reynolds number, *Re*) have been obtained and compared with the literature correlations and the model with a linear variation of λ_{er} in the vicinity of the wall.

2. Experimental

The temperatures of the air flowing through packed beds were measured in steady state experiments. Experimental set-up for determining the heat transfer parameters in the packed bed (Fig. 1) consisted of three tube sections with inner diameter of 84 mm: heating section (330 mm height), calming section (150 mm height) and cooled test section (650 mm height).

The test section was cooled by water flowing through the annular jacket surrounding the inner tube. Water flow rate of about 25 l min^{-1} provided almost constant temperature of the tube wall of the test section (temperature difference along the whole wall was less than 0.5 °C).

The temperature of the air after the heating section reached 90–100 °C. The heating section and the calming section were filled with 4–5 mm porcelain balls to obtain more or less uniform radial temperature and velocity profiles.

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Nomenclature

a	parameter of the SDM, $a = \lambda_{er}/Gc_pR$
a_0	specific surface of one grain without account of channels (m^{-1})
Bi	Biot number, $Bi = \alpha_w R/\lambda_{er}$
c_p	specific heat of the fluid at constant pressure ($J\,kg^{-1}\,K^{-1}$)
d_{eqv}	equivalent hydraulic diameter of the packed bed (m), Eq. (8)
d_{hole}	characteristic length of the hole for Rashig ring (m), Eq. (24)
d_p	diameter of sphere with volume equal to the volume of the grain (m)
D	reactor tube diameter (m)
F_{form}	form factor
G	mass flow rate ($kg\,m^{-2}\,s^{-1}$), $G = u_0\rho_f$
J_0	zeroth-order Bessel function, first kind
J_1	first-order Bessel function, first kind
K	convective heat transport parameter defined by Eq. (20)
l	length of cylindrical particles (m)
Nu_w	Nusselt number, $Nu_w = \alpha_w d_{eqv}/\lambda_f$
Nu_w^*	Nusselt number, $Nu_w^* = \alpha_w d_p/\lambda_f$
Pr	Prandtl number, $Pr = \eta c_p/\lambda_f$
q_R	heat flux from the wall ($J\,m^{-2}\,s^{-1}$)
r	radial co-ordinate (m)
R	reactor tube radius (m)
Re	Reynolds number, $Re = u_0 d_p\rho_f/\eta$
T	fluid temperature (K or °C)
T_w	wall temperature (K or °C)
u_0	superficial fluid velocity ($m\,s^{-1}$)
x	axial co-ordinate (m)
X	mixing length (m), Eqs. (22) and (23)
z	dimensionless axial co-ordinate, $z = x/R$

Greek letters

α_w	wall heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
δ	thickness of the wall-region with low fluid mixing (m)
ε_{bed}	bed porosity without account of porosity of grains
ε_{hole}	porosity of one grain
ε_{hbed}	bed porosity, $\varepsilon_{hbed} = \varepsilon_{bed} + \varepsilon_{hole}(1 - \varepsilon_{bed})$
η	fluid viscosity (Pa s)
λ_{bed}	effective radial thermal conductivity of the bed with a stagnant fluid ($W\,m^{-1}\,K^{-1}$)
λ_{er}	effective radial thermal conductivity of the bed ($W\,m^{-1}\,K^{-1}$)
λ_f	fluid thermal conductivity ($W\,m^{-1}\,K^{-1}$)
μ_n	nth root of characteristic equation, Eq. (6)
ρ	dimensionless radial co-ordinate, $\rho = r/R$
ρ_f	fluid density ($kg\,m^{-3}$)

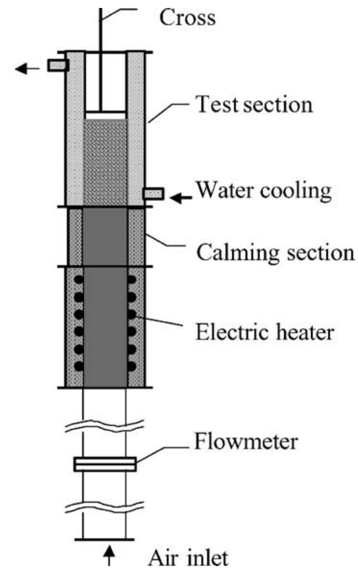

Fig. 1. Experimental set-up.

Air flow rates were determined by measuring the pressure drop on a diaphragm. Superficial velocities of the air were in the range 0.2–2.0 $m\,s^{-1}$. Temperature fields of the air stream issuing from the top of the packing were measured by means of twelve 0.2 mm thermocouples. The latter were held by a glass laminate cross (three thermocouples on each of four arms) in 12 different radial positions. To move the cross easily inside the tube each arm was made 1 mm shorter than the tube radius. A vertical metal rod was attached to the cross centre perpendicular to the plane of the cross. The cross was pressed against the tube wall with two its arms to provide a constant distance from thermocouples to the tube wall. With this design it was possible to turn the cross inside the tube and to fix the immersion depth. The tips of the thermocouples protruded 4–5 mm out of the arms.

At each bed height and air velocity the cross was rotated stepwise full turn with so small angular intervals as necessary to accurately obtain the radial temperature profiles

Table 1
Particles used in experiments

Number	Description	Bed porosity
Spherical particles		
1	Steel, diameter 16 mm	0.41
2	Glass, diameter 19 mm	0.42
Cylindrical particles		
3	Ceramic, diameter 10 mm, length 10 mm	0.40
4	Ceramic, diameter 19 mm, length 19 mm	0.42
Rashig rings		
5	Ceramic, outer diameter 14 mm, length 14 mm, wall thickness 3.5 mm	0.40[a]
6	Copper, outer diameter 14 mm, length 14 mm, wall thickness 1 mm	0.40[a]

[a] Bed porosity without account of porosity of grains.

averaged with respect to the angular co-ordinate. It was determined experimentally that 30° angular intervals or smaller were sufficient for the accurate measurements. The inlet radial temperature profile was measured in the same manner and was used as a boundary condition for the SDM [9].

Details of the packings used in this work are given in Table 1. Bed porosities were measured by a weighting method.

3. Standard dispersion model (SDM)

For the description of the steady state gas temperature profiles in the packed bed the SDM was used:

$$Gc_p\frac{\partial T}{\partial x} = \lambda_{\text{er}}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] \quad (1)$$

$$x = 0, \quad 0 \le r < R, \quad T = f(r) \quad (2)$$

$$r = 0, \quad 0 \le x \le L, \quad \frac{\partial T}{\partial r} = 0 \quad (3)$$

$$r = R, \quad 0 < x \le L, \quad -\lambda_{\text{er}}\frac{\partial T}{\partial r} = q_{\text{R}} = \alpha_{\text{w}}(T - T_{\text{w}}) \quad (4)$$

where $f(r)$ is the inlet temperature profile, and q_{R} is the heat flux to the wall. Wall temperature of the test section T_{w} was considered to be constant. The solution of Eq. (1) with boundary conditions (2)–(4) is:

$$T(\rho, z) = T_{\text{w}} + 2\sum_{n=1}^{\infty}\frac{J_0(\mu_n\rho)\mu_n^2}{[J_0(\mu_n)]^2\{\mu_n^2 + Bi^2\}}\,\mathrm{e}^{(a\mu_n^2 z)} \times \int_0^1 \rho[f(\rho) - T_{\text{w}}]J_0(\mu_n\rho)\,\mathrm{d}\rho \quad (5)$$

Here, $a = \lambda_{\text{er}}/Gc_pR$, $Bi = \alpha_{\text{w}}R/\lambda_{\text{er}}$ and μ_n is the nth ($n = 1, 2, \dots$) positive root of:

$$\mu J_1(\mu) = Bi\, J_0(\mu) \quad (6)$$

The solution (5) contains two parameters: a and the Biot number Bi. Their values determine the effective radial thermal conductivity λ_{er} and the wall heat transfer coefficient α_{w}.

The values of a and Bi were estimated by fitting the solution (5) to the measured temperatures using the method of least squares.

Physical properties of the air were considered to be constant in the whole test section.

4. Relation between the SDM and $\lambda_{\text{er}}(r)$-model

The SDM describes experimental temperature profiles very well in the bed core, but it fails to fit experimental points near the wall at a distance of about particle size δ or less from the wall. Fig. 2 illustrates this for the bed of ceramic cylindrical particles with diameter and length equal to 19 mm.

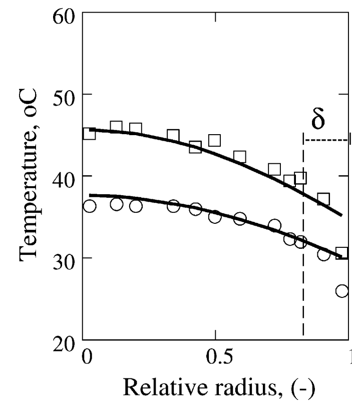

Fig. 2. Temperature profiles after bed of ceramic cylindrical particles with diameter and length equal to 19 mm, Reynolds number 1130, bed heights 100 and 200 mm. Points—experimental data; lines—SDM-profiles.

The drawback of the SDM can be explained by the fact that in the packed bed core the effective radial thermal conductivity $\lambda_{\text{er,core}}$ does not depend on radial position but further near the wall it decreases very fast:

$$\lambda_{\text{er}}(r) = \begin{cases} \lambda_{\text{er,core}} & \text{for } 0 \le r \le R - \delta \\ \lambda_{\text{er},\delta}(r) & \text{for } R - \delta \le r < R \end{cases} \quad (7)$$

where $\lambda_{\text{er},\delta}(R - \delta) = \lambda_{\text{er,core}}$ and $\lambda_{\text{er},\delta}(R) \approx \lambda_{\text{f}}$. Such dependence is due to the mechanism of radial thermal conductivity [4]. It has a convective nature in more or less uniform bed core and at high fluid flow rates. The intensity of fluid mixing gradually decreases in the wall-region where the bed void fraction approaches unity. Dramatic changes in the local porosity in filled tubes had been observed within the distance comparable with the equivalent hydraulic diameter of the packed bed from the wall [3]. After this distance the local porosity practically not changes and can be supposed as constant bed core porosity. Thus, the equivalent hydraulic diameter of the packed bed

$$d_{\text{eqv}} = \frac{4\varepsilon_{\text{bed}}}{a_0(1 - \varepsilon_{\text{bed}})} \quad (8)$$

was chosen as the characteristic thickness of the wall-region.

In this work, the experimental heat transport parameters were determined using only the temperatures measured in the bed core or at the distance larger than $\delta = d_{\text{eqv}}$ from the wall. In that way $\lambda_{\text{er,core}}$ and α_{w} were determined from the experimental temperature profiles for all types of particles.

The wall heat transfer coefficient is defined from Eq. (4):

$$\alpha_{\text{w}} = \frac{q_{\text{R}}}{T_{r=R}^{\text{SDM}} - T_{\text{w}}} \quad (9)$$

where $T_{r=R}^{\text{SDM}}$ is the gas temperature at the wall calculated from the SDM.

Let us assume that the heat flux in the δ-layer near the wall does not depend on radial position, and the fluid temperature at the wall is T_w (boundary condition of the first kind at the wall). In this case, the temperature jump $T_{r=R}^{SDM} - T_w$ predicted by the SDM can be calculated if $\lambda_{er,\delta}(r)$ is known. This can be done as follows.

According to the SDM:

$$q_R = -\lambda_{er,core}\frac{\partial T}{\partial r}, \quad R-\delta \le r \le R \tag{10}$$

and

$$T_{r=R}^{SDM} - T_{r=R-\delta} = -q_R\frac{\delta}{\lambda_{er,core}} \tag{11}$$

According to the $\lambda_{er}(r)$-model:

$$q_R = -\lambda_{er,\delta}(r)\frac{\partial T}{\partial r}, \quad R-\delta \le r \le R \tag{12}$$

and

$$T_w - T_{r=R-\delta} = -q_R\int_{R-\delta}^{R}\frac{dr}{\lambda_{er,\delta}(r)} \tag{13}$$

From Eqs. (11) and (13) it follows:

$$T_{r=R}^{SDM} - T_w = q_R\int_{R-\delta}^{R}\frac{dr}{\lambda_{er,\delta}(r)} - q_R\frac{\delta}{\lambda_{er,core}} \tag{14}$$

Substituting Eq. (14) into Eq. (9), we find the relation between α_w and $\lambda_{er,\delta}(r)$:

$$\alpha_w = \frac{1}{\int_{R-\delta}^{R}[dr/\lambda_{er,\delta}(r)] - (\delta/\lambda_{er,core})} \tag{15}$$

Note that α_w has physical significance only as a parameter, which defines the heat flux from the packed bed to the wall. But the obtained from the SDM value of α_w can be used as a criterion for experimental verification of a chosen function $\lambda_{er,\delta}(r)$.

5. Model with a linear variation of λ_{er} in the vicinity of the wall: correlation between Nu_w and $\lambda_{er,core}$

The correlation between the wall Nusselt number

$$Nu_w = \frac{\alpha_w\delta}{\lambda_f} = \frac{\alpha_w d_{eqv}}{\lambda_f} \tag{16}$$

and $\lambda_{er,core}$ was obtained in assumptions, that:

1. $\lambda_{er,core} \gg \lambda_f$.
2. $\lambda_{er,\delta}(r)$ changes linearly in the wall-region from the fluid thermal conductivity λ_f at the wall to the effective radial thermal conductivity $\lambda_{er,core}$ in the bed core:

$$\lambda_{er,\delta}(r) = \lambda_f + C(R-r) \quad \text{for } R-\delta \le r < R \tag{17}$$

where

$$C = \frac{\lambda_{er,core} - \lambda_f}{\delta} \approx \frac{\lambda_{er,core}}{\delta}$$

After that Eq. (15) can be transformed to

$$\alpha_w = \frac{1}{(1/C)\ln(\lambda_{er,core}/\lambda_f) - (\delta/\lambda_{er,core})} \approx \frac{\lambda_{er,core}}{\delta[\ln(\lambda_{er,core}/\lambda_f) - 1]} \tag{18}$$

And Eq. (16) becomes:

$$Nu_w = \frac{\lambda_{er,core}^*}{\ln\lambda_{er,core}^* - 1} \tag{19}$$

where $\lambda_{er,core}^* = \lambda_{er,core}/\lambda_f$.

6. Results and discussion

At low temperature the general correlation for the effective radial thermal conductivity of the packed bed is commonly written in the form:

$$\lambda_{er,core}^* = \lambda_{bed}^* + KRePr \tag{20}$$

where $\lambda_{er,core}^* = \lambda_{er,core}/\lambda_f$, $\lambda_{bed}^* = \lambda_{bed}/\lambda_f$, and K is the convective heat transport parameter.

Equation of Bauer and Schlunder for the convective heat transport parameter [4] is widely used:

$$K_{Bauer} = \frac{X}{8[2-\{1-(2/(D/d_p))\}^2]d_p} \tag{21}$$

where the mixing length X for cylindrical particles is

$$X_{cyl} = \frac{\varepsilon_{bed}}{\varepsilon_{hbed}}F_{cyl}d_p + \frac{\varepsilon_{hole}(1-\varepsilon_{bed})}{\varepsilon_{hbed}}F_{hole}d_{hole} \tag{22}$$

and the mixing length for spherical particles without holes is

$$X_{sph} = F_{sph}d_p \tag{23}$$

Characteristic length of the hole for Rashig ring

$$d_{hole} = \sqrt{2}l \tag{24}$$

where $F_{sph} = 1.15$, $F_{cyl} = 1.75$, $F_{hole} = 2.8$ are the form factors.

Table 2 presents the values of K_{Bauer} calculated according to the Eq. (21) and the values of $K_{experimental}$, which satisfy the Eq. (20) for experimental dependence of SDM-parameter $\lambda_{er,core}^*$ on Reynolds number. It was estimated that $\lambda_{bed}^* = 10$ for ceramic and glass particles and $\lambda_{bed}^* = 20$ for metallic particles. Values of K_{Bauer} and $K_{experimental}$ are very close. Maximal deviation of 13% was found for ceramic cylinders

Table 2
Convective heat transfer parameter K, Eq. (20)

	Particle type number according to Table 1					
	1	2	3	4	5	6
K_{Bauer}	0.088	0.084	0.147	0.124	0.172	0.206
$K_{experimental}$	0.089	0.091	0.146	0.14	0.16	0.21

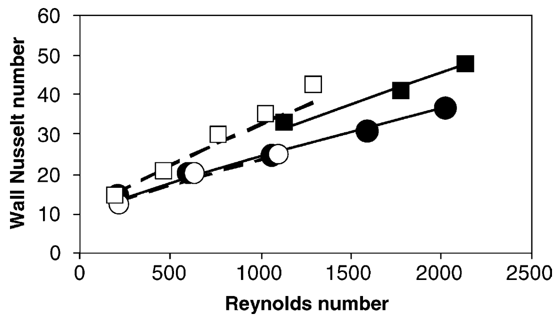

Fig. 3. Dependences of wall Nusselt number on Reynolds number for particles of regular shape. Open circle—glass spheres, diameter 19 mm. Solid circle—steel spheres, diameter 16 mm. Open square—ceramic cylinders, diameter 10 mm, length 10 mm. Solid square—ceramic cylinders, diameter 19 mm, length 19 mm. Lines—Eq. (19).

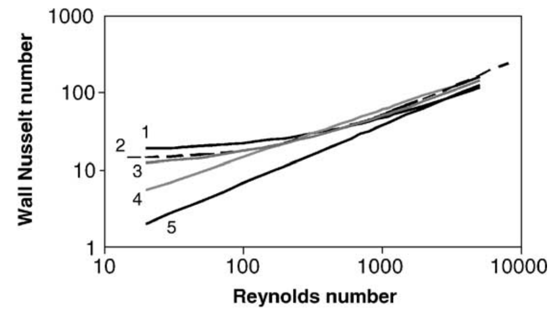

Fig. 5. Comparison of the correlation (26) with recommended in literature correlations between wall Nusselt number and Reynolds number: (1) Martin and Niles [5]; (2) Eq. (26) for spheres; (3) Yagi and Kunii [6]; (4) Dixon and Labua [7]; (5) Kunii and Suzuki [8].

with diameter and length equal to 19 mm. The accuracy of experimental determination of $K_{\mathrm{experimental}}$ was about 10% for spheres and 15% for cylinders and Rashig rings.

Figs. 3 and 4 present Nu_{w} as a function of Re for regular and shaped particles respectively. Theoretical values of Nu_{w} were calculated by using Eq. (19). In that case $\lambda^*_{\mathrm{er,core}}$ was calculated from the model of Bauer and Schlunder (Eqs. (20)–(24)). Experimental points were calculated from values of SDM-parameter α_{w} by using Eq. (16). The Reynolds number is based on the effective particle diameter d_{p}. In case of the Rashig rings the hole was included in the particle volume.

The obtained correlation (19) for the wall Nusselt number is in a good agreement with the experimental data.

Fig. 5 illustrates correlations between Nu_{w} and Re recommended in literature. The wall Nusselt number in these correlations is based on the effective particle diameter:

$$Nu^*_{\mathrm{w}} = \frac{\alpha_{\mathrm{w}} d_{\mathrm{p}}}{\lambda_{\mathrm{f}}} \tag{25}$$

Accordingly Eq. (19) was rewritten to be plotted in Fig. 5:

$$Nu^*_{\mathrm{w}} = \frac{\lambda^*_{\mathrm{er,core}}}{\ln \lambda^*_{\mathrm{er,core}} - 1} \frac{d_{\mathrm{p}}}{d_{\mathrm{eqv}}} \tag{26}$$

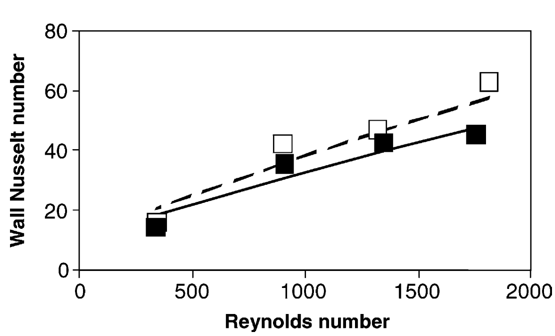

Fig. 4. Dependences of wall Nusselt number on Reynolds number for Rashig rings. Open square—copper rings, diameter 14 mm, length 14 mm, wall thickness 1 mm. Solid square—ceramic rings, diameter 14 mm, length 14 mm, wall thickness 3.5 mm. Lines—Eq. (19).

The correlation (26) gives satisfactory results and coincides well with the recommended correlations at large Reynolds numbers.

7. Conclusions

The $\lambda_{\mathrm{er}}(r)$-model (7) was proposed to the description of the radial heat transfer in the packed bed. The model allows correlation between the SDM-parameters. It results in the simple correlation (19) between the wall Nusselt number and the bed core effective radial thermal conductivity in case of the model with a linear variation of λ_{er} in the vicinity of the wall (17). The experimental data of bed core radial thermal conductivity was found in agreement with a widely used model of Bauer and Schlunder [4]. This classical model describes the effective radial thermal conductivity in the bed core for beds of regular shape particles and Rashig rings, and for different tube to particle diameter ratio D/d_{p}. Comparison of the experimental Nu_{w} with correlation (19) was made when $\lambda^*_{\mathrm{er,core}}$ is calculated from the model of Bauer and Schlunder. The obtained correlation (19) stands comparison with the experimental and literature data.

The model with a linear variation of λ_{er} in the vicinity of the wall does not require any additional empirical parameters for the description of heat transfer in packed beds of shaped particles. The characteristic thickness of the wall-region $\delta = d_{\mathrm{eqv}}$ is defined by means of the bed porosity and the specific surface of the single particle.

Thus, the model with a linear variation of λ_{er} in the vicinity of the wall was verified for beds of spheres, cylinders and Rashig rings.

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